Budget Allocation Across Media Channels

The problem: allocate budget among media channels with the intention of optimizing KPI.

This problem is made harder because you have to take into account multi-touch attribution (aka someone saw the ad on tv then saw it on social media and then looked it up on search before finally converting and generating a sale). I initially thought I'd tackle this problem using statistics, but along the way, statistics made no sense to me. I opted to focus heavily on linear programming as it made more logical sense.

For the attribution model, I realize that this is going to be a very terrible method. I have two metrics I'm looking at:

- Impressions
- KPI

Why am I looking at only these two metrics? I'm going to see the linear relationship between Media Channel A and Media Channel B. I'm also going to make a new metric: Dropouts = Impressions – KPI. Dropouts will represent the pull of people that saw our ad and decided "your ad is cool, but I'm not interested in purchasing anything," Let's look at the correlation between A's Dropouts and B's Impressions.

$$\text{Correlation (r)}= \frac{1}{n-1}\sum\frac{(x-\overline{x})}{s_x}\frac{(y-\overline{y})}{s_y}$$

$$\text{Where } s_x=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$

Where correlation ≥ 0.

I needed something that would give me a value that's bounded by 1. Technically, if you want a really good multi-touch attribution model, you'd want to use probabilities and instead of a direct relationship, you'd want to convey from Channel A, the probability of stopping at Channel B until going to Channel C and generating a sale or something of the sort. Let's call our KPI in this case leads.

Now, let's use the correlation into a correlation matrix.

Let $x_{i,j}$ denote the correlation x of the first channel's i Impressions and the second channel's j Dropouts. If i=j, $x_{i,j}=0$.

Correlation matrix

$$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \end{bmatrix}$$

Let's multiply the correlation matrix with leads. Let x_i denote the leads x of channel i.

Leads matrix

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$[Correlation\ matrix] \times [Leads\ matrix] = [Added\ Value\ matrix]$$

The Added Value matrix will account for those leads from Channel I that can possibly be attributed to Channel j.

$$(Added\ value\ leads) + (Leads) = (Leads\ Significance)$$

$$\frac{k(Channel\ A\ Leads\ Significance + Channel\ B\ Leads\ Significance + Channel\ C\ Leads\ Significance)}{Total\ Leads} = 1$$

Solve for k to get the scalar multiplier to be attributed to each channel's lead significance.

Our main allocation optimization model will be the simplex method.

Let x_i denote the amount allocated x to channel i.

The 100,000 is to help generalize. The output will be a hard number that you can use the 100,000 to give you a percentage of the budget that should go to this channel.

$$x_1 + x_2 + x_3 \leq 100{,}000$$

This will be your optimization maximization equation for the simplex method.

$$\frac{k(Channel\ 1\ Leads\ Significance)}{Channel\ 1\ Spend}x_1 + \frac{k(Channel\ 2\ Leads\ Significance)}{Channel\ 2\ Spend}x_2 + \frac{k(Channel\ 3\ Leads\ Significance)}{Channel\ 3\ Spend}x_3 = Leads$$

You can add in some more constraints depending on how your company wants to allocate budget. But simply put the above equation into the simplex method matrix, solve and voila.